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Section 4.4 Applications of Trigonometry

1. Solve the following trigonometric equations for θ; $0 \le \theta \le 2\pi$: Write a general formula for the solution:

a) $4 = 2\sin\left(\underbrace{2\pi\frac{t-3}{6}}_{=\theta}\right) + 3$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}, \frac{5\pi}{6}$

$2\cancel{\pi}\left(\frac{t-3}{6}\right) = \frac{\cancel{\pi}}{6}$ $\qquad$ $2\cancel{\pi}\left(\frac{t-3}{6}\right) = \frac{5\cancel{\pi}}{6}$

$t = 3.5^R$ $\qquad$ $t = 5.5^R$

Period: $\frac{2\pi}{\frac{\pi}{3}} = 6$

$\boxed{t = 3.5^R, 5.5^R}$

$\boxed{t = 3.5 + 6n^R, 5.5 + 6n^R ; n \in \mathbb{Z}}$

b) $-4.1 = 3\cos\left(\underbrace{2\pi\frac{t-2}{5}}_{=\theta}\right) - 5$

$\cos\theta = \frac{0.9}{3}$

$\theta = 2\pi\left(\frac{t-2}{5}\right) = \cos^{-1}\left(\frac{0.9}{3}\right) = 1.266, 5.017$

$\boxed{t = 3.008}$ $\qquad$ $\boxed{t = 5.992}$

$p = \frac{2\pi}{B} = \frac{2\pi}{\frac{2}{5}\pi} = 5^R$

$\boxed{t = 3.008^R + 5n , 5.992^R + 5n ; n \in \mathbb{Z}}$

c) $3 = 1.2\sin\left(\underbrace{2\pi\frac{t-300}{12}}_{=\theta}\right) + 3.5$

$\sin\theta = \frac{-0.5}{1.2}$

$\theta = \frac{\pi}{6}(t-300) = \sin^{-1}\left(\frac{-0.5}{1.2}\right) = -0.43, 3.57$

$t = 299.18^R$ $\qquad$ $t = 306.82^R$

Period: $\frac{2\pi}{\frac{\pi}{6}} = 12^R$

No roots within $0 \le t \le 2\pi$

$\boxed{t = 299.18^R + 12n, 306.82^R + 12n ; n \in \mathbb{Z}}$

d) $-1.5 = -12.2\cos\left(\underbrace{2\pi\frac{2t-120}{24}}_{=\theta}\right) + 8.5$

$\cos\theta = \frac{10}{12.2}$

$\theta = \frac{\pi}{12}(2t-120) = \cos^{-1}\left(\frac{10}{12.2}\right) = 0.61^R, 5.67^R$

$t = 61.16^R, 70.84^R$

Period: $\frac{2\pi}{\frac{\pi}{6}} = 12^R$

No roots with $0 \le t \le 2\pi$

$\boxed{t = 61.16^R + 12n, 70.84^R + 12n ; n \in \mathbb{Z}}$

2. The water depth at the Vancouver port is between 18m to 24m high. High tide occurs at at 7am. The next time high tide occurs again is 12 hours later.

a. Write a sinusoidal function where the depth of the water is a function of time "t"

Period: 12 $\quad B = \frac{2\pi}{P} = \frac{2\pi}{12} = \frac{\pi}{6}$

V.S.: $\frac{24+18}{2} = 19$

Amplitude: $\frac{24-18}{2} = 3$

$$\boxed{y = 3\cos\left[\frac{\pi}{6}(t-7)\right] + 19}$$

b. Find the depth of the water at 5pm

$$y = 3\cos\left[\frac{\pi}{6}(17-7)\right] + 19 = \boxed{20.5\text{ m}}$$

c. Write a general formula for the time when high tide will occur

$$\boxed{t = 7 + 12n \; ; \quad n \in \mathbb{Z}}$$

d. Write a general formula for the time when low tide will occur

$$\boxed{t = 13 + 12n \; ; \quad n \in \mathbb{Z}}$$

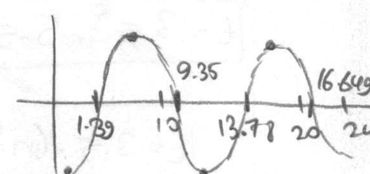
e. If a large shipping freight requires a draft of 20meters, at what times during the day can the freight dock?

$$20 = 3\cos\left[\underbrace{\frac{\pi}{6}(t-7)}_{=\theta}\right] + 19$$

$t_1 = 9.35 \quad t_3 = 13.78$

$t_2 = 16.649 \quad t_4 = 1.39$

$$\cos\theta = \frac{1}{3} \Rightarrow \theta = \frac{\pi}{6}(t-7) = \cos^{-1}\left(\frac{1}{3}\right) = 1.23, 5.05$$



Between 1am - 9am and 13am - 16am

3. A Ferris wheel has a radius of 30m and its center is 33m above the ground. It rotates once every 40s. Suppose you get on at the bottom when t=0.

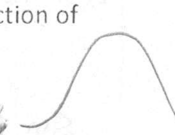
a. Write a sine function and cosine function that describes your height above ground as a function of time

Amplitude: $\frac{30}{2} = 30$

V.S: 33

Period = 40 $\Rightarrow B = \frac{2\pi}{40} = \frac{\pi}{20}$

$$\boxed{\begin{aligned} y &= -30\cos\left(\frac{\pi}{20}t\right) + 33 \\ y &= 30\sin\left[\frac{\pi}{20}(t-10)\right] + 33 \end{aligned}}$$



b. Suppose a passenger will pass out if once they are over a height of 60m. After how many seconds will this passenger pass out?

$$-30\cos\left(\underbrace{\frac{\pi}{20}t}_{=\theta}\right) + 33 = 60$$

$$\theta = \frac{\pi}{20}t = \cos^{-1}\left(\frac{27}{-30}\right) = 2.69^R, 3.59^R$$

$t_1 = 17.13^R$

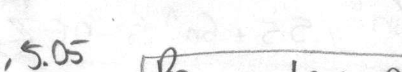
$t_2 = 22.87^R$

For $t_2 - t_1 = 5.7$ seconds

After 17.13 seconds

c. How high will you be after 10s?

$$y = -30\cos\left(\frac{\pi}{20}\cdot 10\right) + 33 = \boxed{33\text{m}}$$



d. For what range in time will your height be above 55m?

$$-30\cos\left(\frac{\pi}{20}t\right) + 33 = 55$$

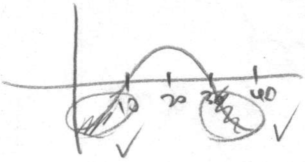
$$\frac{\pi}{20}t = \cos^{-1}\left(\frac{22}{-30}\right) \Rightarrow t = 15.24\text{s} \quad t = 24.76\text{s}$$

$$\boxed{15.24 < t < 24.76}$$

e. Suppose the ferris stopped and everyone had to jump off while on the ride. Statistically, a person has a good survival rate jumping from 10meters high. Anything higher would be dangerous. What percentage of riders would expect to survive?

$$-30\cos\left(\frac{\pi}{20}t\right)+33=10$$

$$\frac{\pi}{20}t=\cos^{-1}\left(\frac{-23}{-30}\right)=0.697,\ 5.59$$



$t=4.44 \qquad t=35.56$

$$\frac{4.44+(40-35.56)}{40}=\boxed{22.2\%}$$

$0\le t\le 4.44 \qquad 35.56 \le t<40$

4. On December 21 of each year, the sun is closest to the Earth at approximately 147.25 million km. On June 21, the sun is at its greatest distance at approximately 152.23 million km.

a. Write a sinusoidal function where "d" distance is a function of the time "t", number of the day in the year

Period: 365 $\quad B=\frac{2\pi}{360}=\boxed{\frac{\pi}{180}}$

Amplitude: $\frac{152.23-147.25}{2}=\boxed{2.49}$

V.S: $\frac{152.23+147.25}{2}=\boxed{149.74}$

$$\boxed{d=-2.49\cos\left(\frac{\pi}{180}t\right)+149.74}$$

b. How far will the earth be from the ~~moon~~ SUN on October 31?

October 31st = 304th day

$$y=-2.49\cos\left(\frac{\pi}{180}\cdot 304\right)+149.74=\boxed{148.35 \text{ million km}}$$

5. On the nth day of the year, the number of hours of daylight in Vancouver is given by the formula

$$h=3.98\sin 2\pi\left(\frac{n-80}{365}\right)+12.158$$

i) How many hours of daylight should there by on June 20th ?

June 20th: 171th day

$$h=3.98\sin\left[2\pi\left(\frac{171-80}{365}\right)\right]+12.158=\boxed{16.14}$$

ii) How many hours of daylight should there by on December 21st?

December 21st = 355th day

$$h=3.98\sin\left[2\pi\left(\frac{355-80}{365}\right)\right]+12.158=\boxed{8.18 \text{ hours}}$$

iii) Suppose there was an astronomical event that shifted the rotation of the earth, causing the number of days for one cycle around the sun to be decreased by 5 days and reduced the number of sunlight of each day in Vancouver by 2 hours throughout the whole year, what would your new function be?

Period: 360 $\quad B = \frac{2\pi}{360} = \frac{\pi}{180}$ $\qquad$ V.S: $12.158 - 2 = 10.158$

$$\boxed{y = 3.98\sin\left[\frac{\pi}{180}(n-80)\right] + 10.158}$$

6. The pedals of a bicycle are mounted on a bracket whose center is 29cm above ground. Each pedal is 17cm from the bracket. Assume that the bicycle is pedalled at the rate of 13 cycles per minute.

   a. Write an equation for the height of the pedal as a function of time

Amplitude: 17
V.S: 29
Period: $\frac{1}{13} \Rightarrow B = \frac{2\pi}{\frac{1}{13}} = 26\pi$

$$\boxed{y = 17\sin(26\pi t) + 29}$$

   b. If the bicycle is pedalled faster, how will it change the amplitude, period, and phase shift of the function?

Phase shift: remains the same
Amplitude: " " "
Period: compressed (becomes less)

7. Given the function $y = 3\cos\frac{2\pi(t-30)}{240} + 12$, write it as a Sine function

$\cos\theta = \sin\left(\frac{\pi}{2} + \theta\right)$

$$y = 3\sin\left(\frac{2\pi(t-30)}{240} + \frac{\pi}{2}\right) + 12$$

OR Period: $\frac{2\pi}{\frac{2\pi}{240}} = 240^R$

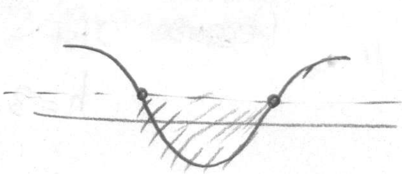
Add to the period

$$\boxed{y = 3\sin\left(\frac{2\pi(t+30)}{240}\right) + 12}$$

b) Solve: $14 > 3\cos\frac{2\pi(t-30)}{240} + 12$

$$3\cos\left[\frac{2\pi}{240}(t-30)\right] + 12 = 14$$

$$\frac{2\pi}{240}(t-30) = \cos^{-1}\left(\frac{2}{3}\right) = 0.84, 5.44$$



$$t = 62.13, \ 237.87$$

Period: $\frac{2\pi}{\frac{2\pi}{240}} = 240^R$

$$\boxed{62.13 + 240n \leq t \leq 237.87 + 240n}$$